

Phase Locked Frequency Generators in the ARIEL e-Linac

Marianne Kobierski, 27475110

30 August 2013

TRIUMF, RF Controls

Under the supervision of Ken Fong

Preface

TRIUMF, Canada's national laboratory for particle and nuclear physics, is constructing a linear electron accelerator (e-linac^{*}) to aid in the research and production of medical isotopes in the Advanced Rare IsotopE Laboratory (ARIEL^{*}) project. The e-linac is powered by two radio frequency^{*} (RF) waves, the first a 650 MHz signal that excites the electron gun; the second a 1.3 GHz signal that excites the e-linac cavities (Figure 1). The two signals should not shift in phase relative to each other. This phase locking^{*} is theoretically achievable since the higher frequency, 1.3 GHz, is an exact multiple of the lower frequency, 650 MHz. However, in practical application signals experience jitter^{*}, which is a time-domain manifestation of the frequency domain phenomenon of phase noise^{*}. If too large, the jitter could disrupt the experimental setup.

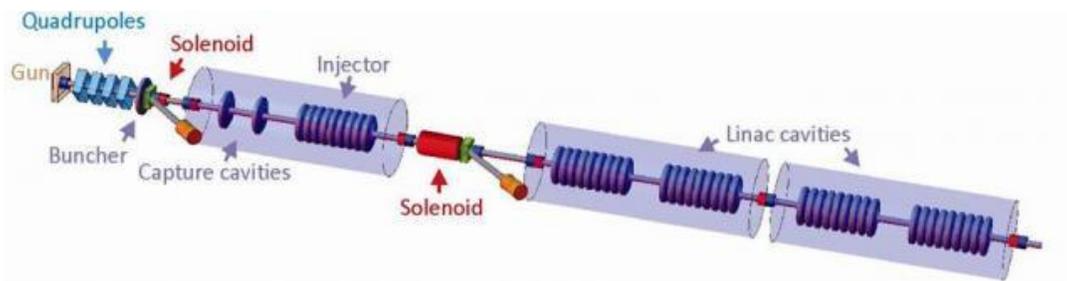


Figure 1: ARIEL e-linac. The gun (far left) is excited by 650 MHz; the Linac cavities (right) are excited by 1.3GHz. (TRIUMF)

Two Rohde & Schwarz SMC100A signal generators will be used in ARIEL. These generators can be phase locked together, tying the phase of the output of one generator to that of the other. If the phase noise between the locked generators is less than 0.1° , a threshold specified by the RF Controls Group Leader, then the generators can be used in the ARIEL project.

This project could not have been completed without the guidance of RF Controls Group Leader Ken Fong and RF Controls Engineering Technician Glen Dennison.

^{*} This and all future terms marked by an asterisk (^{*}) are defined in Appendix F: Glossary

Summary

In order to meet the requirements for providing RF power to the ARIEL e-linac, the maximum expected phase noise between two phase locked Rohde & Schwarz frequency generators must be proven to be less than 0.1 degree. This study was done to determine the maximum phase noise between two phase locked Rohde & Schwarz signal generators (SMA100A and SMC100A), under the assumption that the phase noise between the two SMC100A generators that will be used in the ARIEL e-linac will fall in a similar range.

The maximum expected phase noise was found by generating a voltage curve of noise between the two locked signal generators versus frequency, using a conversion factor to convert noise in volts to noise in degrees, and integrating this curve to find overall maximum phase noise in degrees.

It was determined that the phase noise between the two signal generators was 0.0122 ± 0.0006 degrees. As this falls below the 0.1 degree requirement, it can be concluded that the phase noise associated with phase locking the frequency generators is small enough to be sufficient for use in ARIEL. It is therefore recommended that the two signals for the ARIEL e-linac project be locked together using the phase-locking mechanism built into the Rohde & Schwarz frequency generators.

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Introduction

TRIUMF is currently constructing a linear electron accelerator as part of ARIEL, a project that will contribute to research on and production of medical isotopes. In order to power the e-linac, two frequencies are to be applied to different sections of the linac, but they must be phase locked together such that they never shift relative to each other by more than 0.1 degree.

This report documents the study undertaken to determine the expected phase shift between signals from two Rohde & Schwarz signal generators (models SMA100A and SMC100A) locked together as described in Appendix A: How to Phase Lock two Frequency Generators. The results will be used to decide whether or not phase locking the two frequency generators for ARIEL using this method is acceptable, under the assumption that the phase noise between the two SMC100A generators that will be used in the ARIEL e-linac will fall in a similar range to those being tested in this study.

As illustrated in Figure 2, phase noise was found by generating a voltage curve of noise between the two locked signal generators versus frequency, using a conversion factor to convert noise in volts to noise in degrees, and integrating the converted curve to find overall maximum phase noise in degrees.

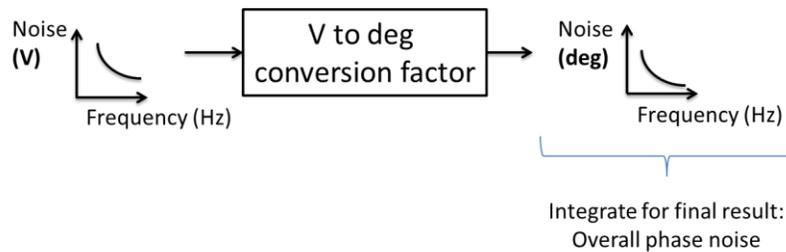


Figure 2: Approach to finding overall phase noise

For this report it is more natural to describe the process in the following order:

1. Using a phase detector* to determine the voltage to degrees conversion factor,
2. Using a phase detector and a dynamic signal analyzer to generate a curve of noise in volts versus frequency in hertz,
3. Processing noise-frequency data to produce an overall phase noise estimate.

The equipment used in this study is as follows:

- One Tektronix TDS 2024 Four Channel Digital Storage Oscilloscope
- One Rhode & Schwarz SMC100A Signal Generator
- One Rhode & Schwarz SMA100A Signal Generator
- One Agilent 35670A Dynamic Signal Analyzer
- One Mini-Circuits ZX90-2-11-S+ Frequency Multiplier
- One Mini-Circuits ZFM-5X-S+ Frequency Mixer
- Coaxial Cables

Discussion

This project depended largely on designing and assembling a phase detector, in order to generate the noise (in $V_{rms}/rtHz$) vs frequency curve, and to produce the conversion factor between volts and degrees. Once the noise vs frequency curve was generated, the data had to be processed to yield the overall phase noise.

Phase Detector

A phase detector is a device that, when provided with two signals of the same frequency, outputs a signal that corresponds to the phase difference* between the two signals (Figure 3).

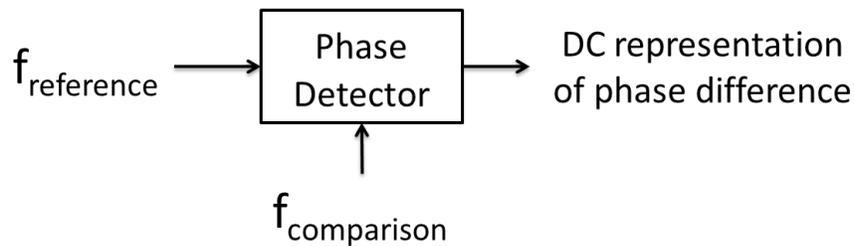


Figure 3: Phase detector block diagram

Phase difference includes both phase noise and phase offset* – the amount by which two signals differ from being in phase (Figure 4).

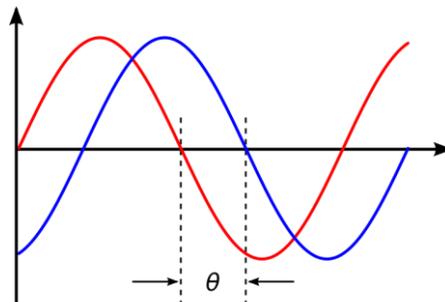


Figure 4: The phase offset between these two signals has a value of θ . (Peppergrower, 2009)

One method for creating a phase detector involves using a double balanced mixer* (DBM), a device that, when provided with two signals of any frequency, outputs a signal that contains the sum and the difference of the two input signals as shown in Figure 5 (Kurtz, 2001) (Sayre, 2008).

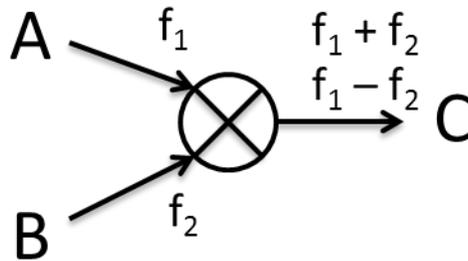


Figure 5: Schematic of a double balanced mixer

The DBM output equation can be written as

$$V_{o/p} = K * [\cos((\omega_1 - \omega_2)t - \varphi) + \cos((\omega_1 + \omega_2)t + \varphi)], \text{ where}$$

- K is a constant,
- ω_1 and ω_2 are the input frequencies, and
- φ is the phase offset between the two input signals.

The full derivation can be found in Appendix B: Double Balanced Mixer Derivation.

In the output voltage equation there is a high frequency term and a low frequency term. The low frequency $\cos((\omega_1 - \omega_2)t - \varphi)$ term contains information about the difference in phase between the two signals. The high frequency $\cos((\omega_1 + \omega_2)t + \varphi)$ term, on the other hand, contains no useful information for this application. It was therefore necessary to remove the high frequency term from the equation in order to gain access to the information held in the low frequency term. This was accomplished by sending the DBM output through a low pass filter. Details of filter parameters and design are documented in Appendix C: Low Pass Filter Design.

With a phase detector composed of a DBM and a low-pass filter, the DBM high frequency term is removed, simplifying the detector output to

$$V_{o/p} = K * [\cos((\omega_1 - \omega_2)t - \phi)].$$

Although the two input frequencies are set at the same value, 1.3 GHz, $(\omega_1 - \omega_2)$ is not exactly equal to zero since the two input frequencies continue to experience some jitter, or phase noise. The output voltage therefore contains both an AC and a DC component. The DC component is directly related to the phase offset between the two input signals (ϕ), while the AC component contains information about the phase noise $(\omega_1 - \omega_2)$.

DC Component – Voltage-to-Degrees Conversion Factor

When only the DC components of the phase detector output equation are considered, the formula simplifies to

$$V_{o/p} = K * [\cos(\phi)].$$

This indicates that a direct relationship between voltage and phase shift exists, and that the relationship is sinusoidal. Since phase shift can only occur in a 180-degree span before repeating, the sinusoid will be bounded such that each phase shift value corresponds to only one voltage value.

In order to find the exact relationship between phase difference and the phase detector output, the two frequency generators were locked together at a specified phase offset as described in Appendix A: How to Phase Lock two Frequency Generators, and an oscilloscope was used to determine the DC phase detector output voltage. Three sets of voltage data were collected and averaged as the phase offset was varied from -90 to +90 degrees. The resulting curve is shown in Figure 6.

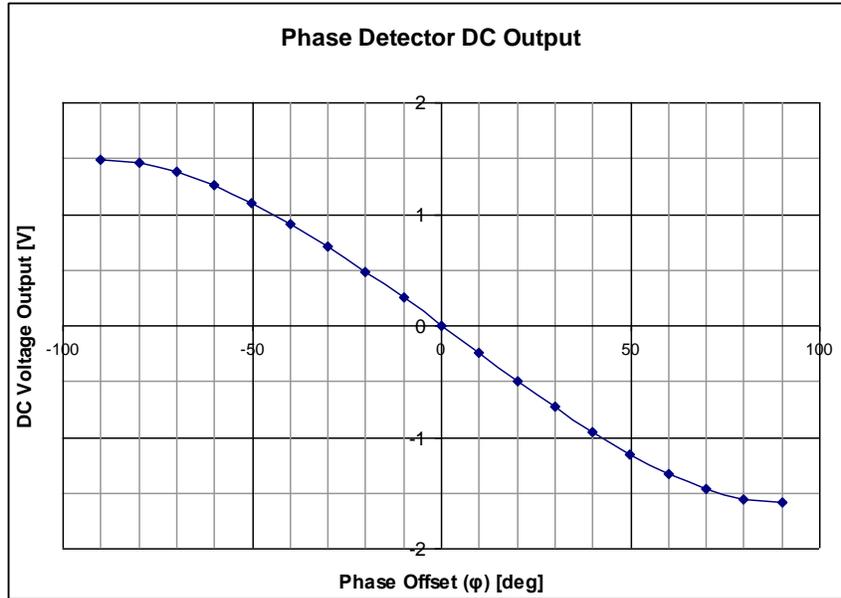


Figure 6: Graph of phase detector DC output

Since this study is investigating the phase jitter between two locked signals, the phase differences between the two signals will be very small. Therefore, only the region where the two signals are in phase, or very close to in phase, is of interest. This occurs around a phase shift of 0 degrees, a region which is effectively linear in this plot. From Figure 7, this linear region has a slope of 0.0242 and the standard error on this conversion value was calculated to be 0.0002.

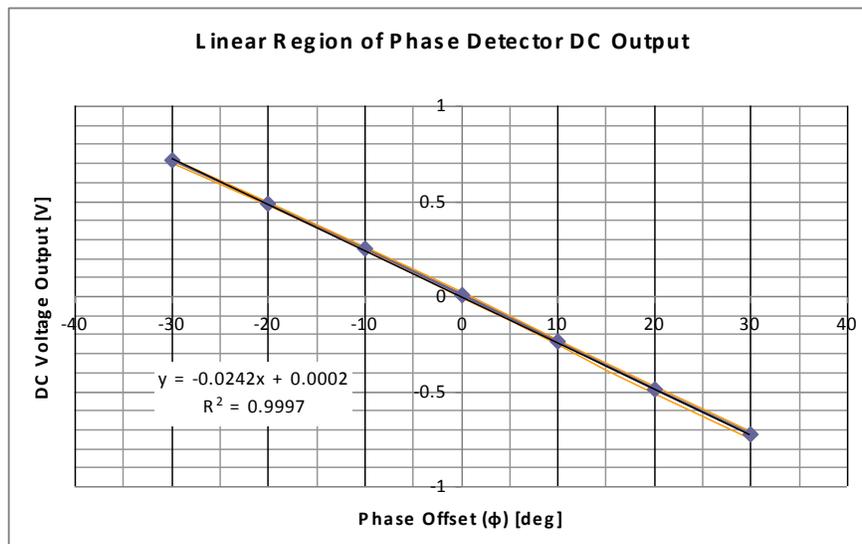


Figure 7: Linear region of phase detector DC output with error bounds based on three trials

The relationship between phase shift in volts and phase shift in degrees is therefore

$$\phi[\text{V}] = 0.0242 * \phi[\text{deg}].$$

This conversion will be used in converting noise voltage data to a final phase noise term in degrees.

AC Component – Phase Noise (In Volts)

In the phase detector output equation,

$$V_{o/p} = K * [\cos((\omega_1 - \omega_2)t - \phi)],$$

the $(\omega_1 - \omega_2)$ term contributes an AC component to the output signal that reflects the phase noise between the two phase detector input signals. To capture this information in a useable format, the output signal from the phase detector was fed into an Agilent 35670A Dynamic Signal Analyzer* in order to produce a graph depicting the magnitude of noise (in $V_{rms}/rtHz$) contributed by each frequency present in the output signal, within a designated span (Figure 8).



Figure 8: Noise [$V/rtHz$] vs Frequency [Hz] plot from signal analyzer

The signal analyzer measures the magnitude of the signal at each frequency within a user-specified frequency range. Around zero Hertz, the analyzer displays the magnitude of any DC signal that might be present in the signal. As the DC component of the signal is irrelevant in

regards to phase noise between the two signal generators, this information was removed as described in Appendix D: Removing the DC Component in the Signal Analyzer Plot.

Since the frequency range reported by the signal analyzer is limited, if the magnitude was displayed in V_{rms} it would be impossible to know what the overall magnitude of the input signal was. In order to retain this information, the user can specify that the magnitude should be displayed in V_{rms} per root Hertz, which yields data that can be processed to determine all the signal magnitude, or phase noise, information regardless of the span of the frequency plot.

Data Processing

The previous section largely described the theory for assembling the experimental apparatus and collecting data to determine phase noise between two frequency generators. Once the data is collected, it must be manipulated to find the value of interest: the phase noise (in degrees) between the two frequency generators.

For finding the phase noise in degrees from a single set of data, the following method should be followed:

1. Delete the first five data points from the data set in order to remove the DC signal influence (Refer to Appendix D: Removing the DC Component in the Signal Analyzer Plot).
2. Square each V_{rms}/\sqrt{Hz} term.
3. Calculate the area under the curve using trapezoidal approximation.
4. Take the square root of the area term.
5. Convert the rooted area term to degrees using $\phi[deg] = \phi[V] / 0.0242$.

As mentioned in section “AC Component – Phase Noise (In Volts)”, the data is collected in V_{rms}/\sqrt{Hz} to retain information about total noise regardless of frequency span length. This

was experimentally proven by calculating noise between two frequency generators at three different frequencies – the results can be found in Appendix E: Analyzer Vrms/rtHz Noise.

Finally, when calculating the phase noise between the two frequency generators it must be noted that the frequency generators are not the only source of noise in this experiment. The setup consisting of the mixer, the low pass filter, and the signal analyzer itself could all be contributing noise. The noise contributed by the setup can be found by splitting the signal from one signal generator and feeding it into both ports of the phase detector. This serves to completely eliminate the noise between the two input signals – jitter in out input is exactly matched by jitter in the other – so the $(\omega_1 - \omega_2)$ term in the phase detector output equation is always exactly zero. Therefore, the noise calculated from this configuration comes only from the setup:

$$N_{g1}^2 = \Delta n_{\text{setup}}^2.$$

N_{g1} was calculated to be 0.00075 ± 0.00004 degrees, based on a total of eighteen sets of data – three trials at each of three different frequency spans on each of the two frequency generators.

Using two frequency generators the calculated noise includes phase noise between the two input signals as well as noise from the setup:

$$N_{g1-g2}^2 = \Delta n_{g1-g2}^2 + \Delta n_{\text{setup}}^2.$$

N_{g1-g2} was calculated to be 0.0122 ± 0.0006 degrees based on a total of nine sets of data – three trials at each of three different frequency spans.

The noise between the two signal generators, Δn_{g1-g2} , could therefore be calculated from

$$\Delta n_{g1-g2} = \text{sqrt}(N_{g1-g2}^2 - N_{g1}^2).$$

This calculation yields a final value of 0.0122 ± 0.0006 degrees phase noise between the two signal generators.

Conclusions

Two Rohde & Schwarz signal generators, a SMA100A and a SMC100A, were locked together using the built-in signal locking mechanism, and the phase noise between the signal generators was calculated using a signal analyzer and phase detector composed of a double balanced mixer and a low pass filter. The phase noise between the signals was determined to be 0.0122 ± 0.0006 degrees. Under the assumption that two SMC100A signal generators will behave similarly, the phase noise between the two SMC100A will fall in in a similar range as the phase noise between the SMA100A and the SMC100A generators.

Recommendations

The RF Controls Group Leader determined that phase noise less than 0.1 degree between the 650 MHz and the 1.3 GHz RF signals is acceptable in the ARIEL e-linac project, but that anything greater would be unacceptable necessitate a different approach. It has been concluded that the phase noise between two SMC100A generators will fall in a range similar to the phase noise between the SMA100A and the SMC100A generators, which was determined to be 0.0122 ± 0.0006 degrees. This is well below the maximum threshold of 0.1 degree. Therefore it is recommended that

1. the two SMC100A generators should be used to provide the RF signals to the ARIEL e-linac, and
2. the generators should be locked together using the signal locking mechanism that is built into the generators.

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Appendix A: How to Phase Lock two Frequency Generators

In order to phase lock two Rohde & Schwarz frequency generators, the following steps should be followed:

1. Connect a coaxial cable from the REF OUT port at the back of one generator, henceforth termed the “reference oscillator*”, to the REF IN port at the back of the other.
2. On the reference oscillator, select the button titled “config...” in the RF box on the main screen to bring up the RF menu. Select “Ref Oscillator...” and ensure that the Source is set to Internal. Hit the “ESC” button to return to the main screen.
3. On the second frequency generator, bring up the RF menu using the same method and set the Source to External. The two frequency generators are now phase locked. Hit the “ESC” button to return to the main screen.
4. To control the phase shift between the locked signals, select “Phase...” from the RF menu and change the value in the Delta Phase field until the desired phase offset is reached.
5. Ensure that the signals are locked together as desired by visually inspecting the signals using an oscilloscope.

Appendix B: Double Balanced Mixer Derivation

A double balanced mixer is an electrical device that produces an output (RF) signal that contains both the sum and the difference of two input signals (LO and IF). The electrical configuration that accomplishes this is shown in Figure 9.

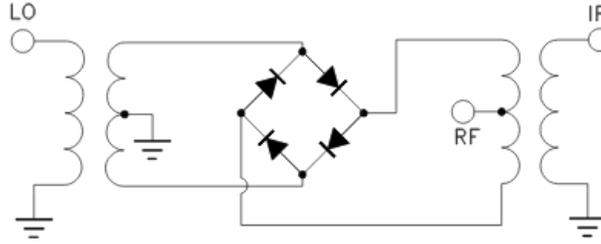


Figure 9: DBM Electrical Schematic. (Coaxial Frequency Mixer)

In this study, the two input signals were sinusoidal. The LO input could therefore be represented by

$$V_{LO}(t) = C_{LO}\cos(\omega_{LO}t),$$

and since the second input signal was set at the same frequency but at a potentially non-zero offset, the IF input could be represented by

$$V_{IF}(t) = C_{IF}\cos(\omega_{IF}t + \varphi(t)).$$

The RF output voltage is equal to the product of the two input voltages:

$$V_{RF}(t) = C * V_{LO}(t) * V_{IF}(t)$$

$$V_{RF}(t) = C * C_{LO}\cos(\omega_{LO}t) * C_{IF}\cos(\omega_{IF}t + \varphi(t))$$

After noting that the phase offset $\varphi(t)$ is time-invariant since the two input signals are locked together, collecting the constants, and using the cosine product-to-sum identity

$$\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B)),$$

the DBM output voltage simplifies to

$$V_{RF}(t) = K * [\cos((\omega_{LO} - \omega_{IF})t - \varphi) + \cos((\omega_{LO} + \omega_{IF})t + \varphi)].$$

Appendix C: Low Pass Filter Design

Since only the low frequency component of the phase detector output voltage signal contains useful information for this study it was necessary to filter out the high frequency component.

In ARIEL, two frequencies are used: 650 MHz and 1.3 GHz. Since a phase detector requires that both input signals have the same frequency, the 650 MHz signal was be doubled using a frequency multiplier. Therefore, the phase detector input signals were both at 1.3 GHz.

Once it was established that the phase detector input frequencies were 1.3 GHz, the high frequency $\cos((\omega_1 + \omega_2)t + \phi)$ term was known to have a frequency of 2.6 GHz. In order to access the phase information contained in the low frequency term, a low-pass filter was created to remove the 2.6 GHz term.

Since the only constraint on the cut-off frequency for the low pass filter was that it must be lower than 2.6 GHz, it was safe to choose a very basic filter design (Figure 10), and to establish the cut-off frequency at 100 kHz.

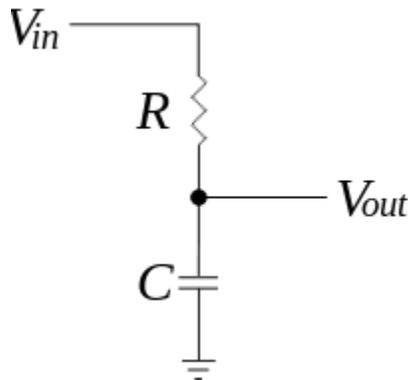


Figure 10: Basic Low Pass Filter Design. (ZooFari, 2009)

Most RF devices have a set impedance* of 50 Ω . This is because part of the RF signal will reflect back towards the source if the impedance of the medium through which it is travelling changes abruptly. The signal generator outputs have an impedance of 50 Ω , as do the

inputs to the mixer. The impedance of the mixer output was unspecified, however, and was therefore determined by creating a voltage divider by placing a resistor between the mixer output and ground. A resistance range of 10 to 20 Ω was found to cause the loaded output voltage to be equal to half of the unloaded output voltage; this range of resistance approximated the value of the mixer output impedance.

For simple low pass filters, the cut-off frequency is described by the formula

$$f_c = 1 / (2 * \pi * R * C).$$

Using

- $f_c = 100$ kHz, and
- $R = 15$ Ω ,

the required capacitance was calculated to be

- $C = 0.1$ μ F.

Appendix D: Removing the DC Component in the Signal Analyzer Plot

The signal analyzer measures the magnitude of the signal at each frequency within a user-specified frequency range. Around zero Hertz, the signal analyzer displays the magnitude of any DC signal that might be present in the analyzer input signal. Since the DC component of the signal is irrelevant in regards to phase noise between the two signal generators, this information had to be removed.

Since the number of data points assigned by the signal analyzer to the DC component is fixed regardless of the frequency span, ten data sets were taken at several different frequencies, and from a visual inspection of the first few data points it was determined that the first four data points had to be removed (Figure 11).

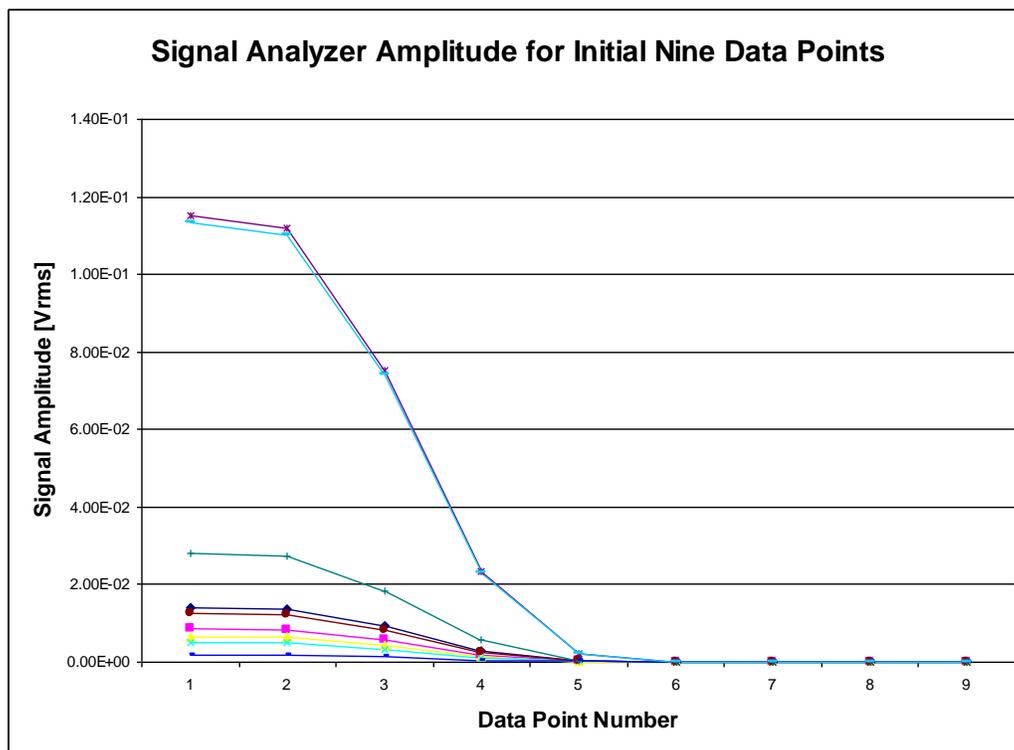


Figure 11: Initial Nine Data Points at Four Different Frequency Spans

After removing the first four data points, the data was re-plotted up to 1.8 kHz (Figure 12). It was evident that the fifth data point had to be removed as well.

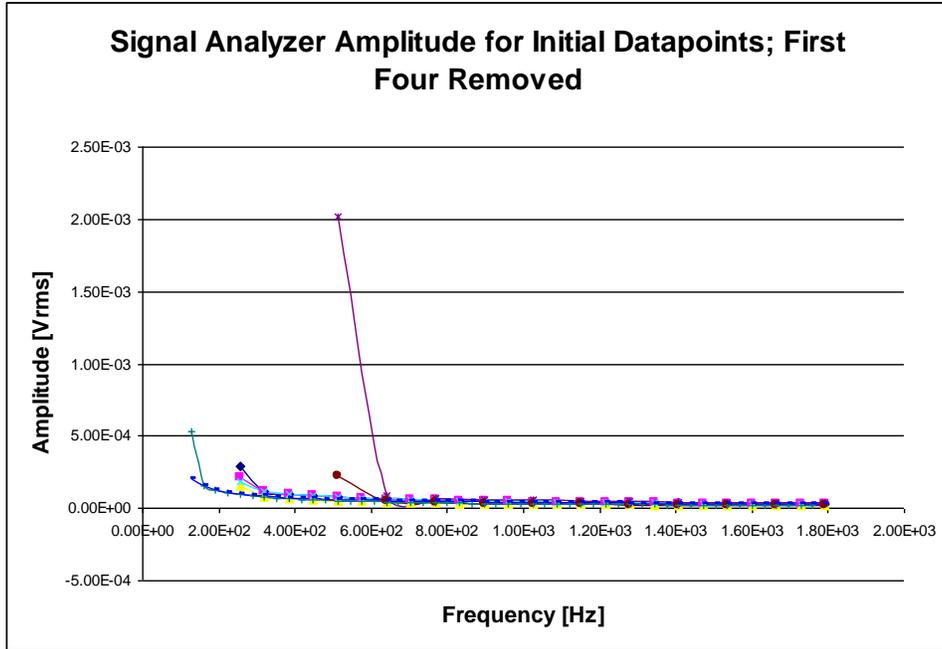


Figure 12: Initial Data Points From Four Different Frequencies, with the Initial Four Points Removed

With the first five data points removed, the DC influence was clearly gone, and only the AC component of the phase detector output voltage remained (Figure 13).

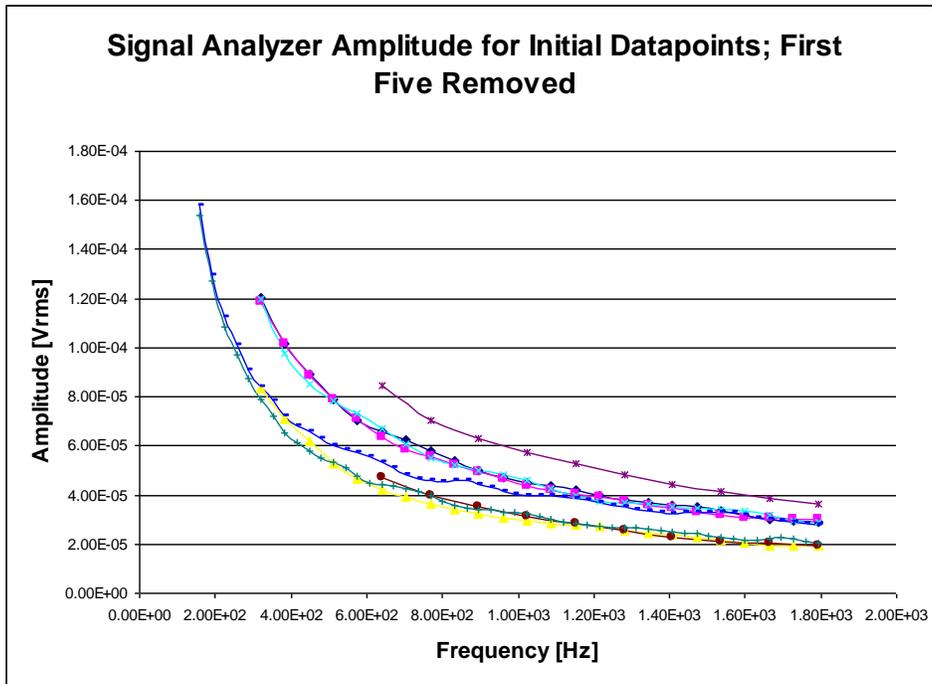


Figure 13: Initial Data Points From Four Different Frequencies, with the Initial Five Points Removed

Appendix E: Analyzer Vrms/rtHz Noise

Data is collected from the signal analyzer in Vrms/rtHz, as this will theoretically retain information about total noise regardless of frequency span length. In order to affirm this, the average noise was calculated at three different frequency spans: 6.25 Hz, 12.5 Hz, and 25 Hz. The resultant noise, each value calculated based on three data sets, is shown in Figure 14.

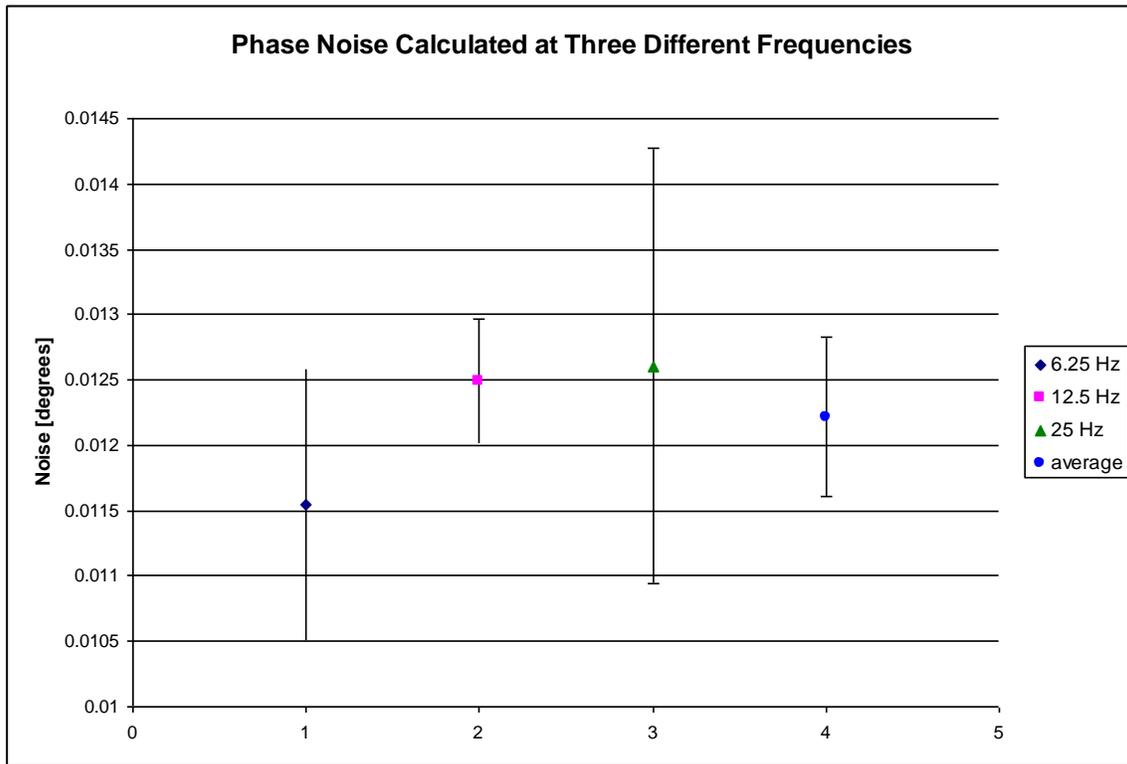


Figure 14: Phase Noise Calculated at Three Different Frequency Spans

Since the three averages fall within an acceptable error range, it is confirmed that the total noise doesn't depend on span.

Appendix F: Glossary

ARIEL	Advanced Rare IsotopE Laboratory: a project designed to aid in the research and production of medical isotopes.
Double Balanced Mixer	A device that outputs a signal that contains the sum and the difference of two input signals
Dynamic Signal Analyzer	A device that displays the magnitude of each frequency component of a signal
E-linac	Linear electron accelerator: a device that accelerates electrons linearly.
Impedance	A measure of how much the flow of current is opposed in a circuit.
Jitter	Deviations from the desired signal frequency, measured in the time domain.
Phase detector	A device that outputs a signal that corresponds to the phase difference between the two signals.
Phase difference	The variation between two signals at the same frequency. This is a combination of phase offset and phase noise.
Phase locked	Signals that oscillate with their phase fixed relative to each other.
Phase noise	Deviations from the desired signal frequency, measured in the frequency domain.
Phase offset	The amount by which two signals differ from being in phase
Radio Frequency	Signals that oscillate at 3 kHz to 300 GHz.
Reference Oscillator	A device that oscillates at a known rate, which provides a benchmark for the oscillation of other devices.